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UNCERTAINTY EVALUATION OF THE PROCESSING ALGORITHM OF A TIME-VARIABLE QUANTITIES IN MULTI-CHANNEL MEASUREMENT SYSTEMS

In this paper the application of the generalised law of uncertainty propagation in determining the standard uncertainty of measurement in a multi-channel sampling transducer has been proposed. It has been shown that for time-variable quantities one of the sources of processing uncertainty is the timing jitter between the successive samples and particular channels of the transducer. The influence of the uncertainty propagation related to the quantization and sampling of time variable signals by the processing algorithm has been analysed. The analysis results have been used for the uncertainty propagation evaluation in an impedance components measurement circuit.

Keywords: uncertainty evaluation, uncertainty propagation, sampling transducer, impedance measurement

1. INTRODUCTION

In measurement circuits with sampling processing the measured value is determined with an appropriate processing algorithm, on the basis of the sequences of N actual values measured directly in P channels of analog-to-digital conversion. Furthermore, depending on the character and range of variability of the measured values in the particular processing channels, analog circuits for signal pre-processing, such as non-electric quantity sensors or signal conditioning systems are used. The processed measurement data are obtained by sampling and quantization of actual values of the measured signals. Both these processes can also be the source of the combined uncertainty of the final processing result [1].

The measurement result obtained by means of such a transducer can be treated as obtained by an indirect method, based on the digitized values measured in the successive N processing and bound up with the function resulting from the realised processing algorithm. Therefore, the form of the realised processing algorithm influences the value of the combined standard uncertainty through the values of the sensitivity coefficients occurring in the equation that describes the uncertainty propagation law [2-4].

In a P-channel sampling transducer the processing algorithm in the general case can be described with the matrix function of the matrix variable $\mathbf{Y} = \mathbf{F}(\mathbf{X})$, as the processing result is either the output vector or the output matrix, while the input matrix can be written as

$$\mathbf{X} = [\mathbf{x}_1 \quad \mathbf{x}_2 \quad \cdots \quad \mathbf{x}_i \quad \cdots \quad \mathbf{x}_P], \tag{1}$$

where $\mathbf{x}_i^{\mathbf{T}} = [x_{1i}, x_{2i}, \ldots, x_{ni}, \ldots, x_{Ni}], \quad i = 1,\ 2,\ ...,\ P$.

In order to describe the combined processing uncertainty of such a transducer, the uncertainty propagation law presented in the Guide [2] should be extended to the case of the matrix function of a matrix variable. Simultaneously, it needs to be taken into account that correlation between the input values may appear and the signal pre-processing circuits have an influence on the value of processing uncertainty.

In this paper the possibility of the application of the generalised law of uncertainty propagation [4] for determining the combined standard uncertainty of a multi-channel sampling transducer has been presented. Examples of the use of this method for evaluation of uncertainty in circuits for the measurement of impedance components have been analysed.

2. GENERALISED LAW OF UNCERTAINTY PROPAGATION

For the scalar output value y bound up with $x_1, x_2, \ldots, x_N$ input values by function f, its estimate $\tilde{y}$ is obtained by the indirect method on the basis of the model given by

$$\tilde{y} = f(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_N). \tag{2}$$

The combined standard uncertainty $u_c(y)$ of estimate $\tilde{y}$, on the grounds of approximation of Eq. (2) by using Taylor series with first-order terms, can be described by the well-known law of propagation of uncertainties [2]

$$u_c^2(y) \approx \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j} u(x_i, x_j) = \sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j} u(x_i, x_j). \tag{3}$$

If the nonlinearity of function f is significant, it is necessary to include in the expression for $u_c^2(y)$, Eq. (3), higher-order terms of the Taylor series expansion [2].

Equation (3) describing the combined standard uncertainty of the scalar value can be generalised to the case of a matrix function of a matrix variable [4]. If $R \times S$ - dimensional real matrix measurand $\tilde{\mathbf{Y}}$ is modelled by

$$\tilde{\mathbf{Y}} = \mathbf{F}(\tilde{\mathbf{X}}) = \begin{bmatrix} f_{11}(\tilde{\mathbf{X}}) & \cdots & f_{1S}(\tilde{\mathbf{X}}) \\ \vdots & \ddots & \vdots \\ f_{R1}(\tilde{\mathbf{X}}) & \cdots & f_{RS}(\tilde{\mathbf{X}}) \end{bmatrix}, \ \tilde{\mathbf{X}} \in \mathbb{R}^{N \times P}, \tag{4}$$

where the estimate of the measured quantity is marked with a tilde, the derivative of this function can be defined using the properties of the vector operator [4-5] which converts $N \times P$ - dimensional matrix $\mathbf{A}$ into $NP \times 1$ - dimensional vector

$$vec\mathbf{A} = \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \vdots \\ \mathbf{a}_P \end{bmatrix}, \quad \text{where } \mathbf{A} = [\mathbf{a}_1 \ \ \mathbf{a}_2 \ \ \cdots \ \ \mathbf{a}_i \ \ \cdots \ \ \mathbf{a}_P], \ \ \mathbf{a}_i^{\mathrm{T}} = [a_{1i} a_{2i}, \ldots, a_{Ni}]. \tag{5}$$

Then, the first derivative of this function with respect to the **X** variable can be written as $RS\times NP$ - dimensional Jacobian matrix [3-5]:

$$\frac{d\mathbf{Y}}{d\mathbf{X}}=\frac{d\mathbf{F}(\mathbf{X})}{d\mathbf{X}}=\frac{\partial vec\mathbf{F}(\mathbf{X})}{\partial(vec\mathbf{X})^{\mathrm{T}}}\,. \tag{6}$$

On the basis of the first-order Taylor approximation in the neighbourhood of the estimate of measured quantity $\tilde{\mathbf{Y}}$ for the vectorial form of this function

$$vec(\mathbf{Y}-\tilde{\mathbf{Y}})=vec(\Delta\mathbf{Y})\approx\frac{d\mathbf{F}}{d\mathbf{X}}vec(\Delta\mathbf{X}) \tag{7}$$

and definition of the covariance matrix

$$\mathbf{C}(\mathbf{Y})=\mathrm{E}\left[\left(vec(\mathbf{Y}-\tilde{\mathbf{Y}})\right)\left(vec(\mathbf{Y}-\tilde{\mathbf{Y}})\right)^{\mathrm{T}}\right]\ \in\mathbb{R}^{RS\times RS}, \tag{8}$$

$$\mathbf{C}(\mathbf{X})=\mathrm{E}\left[\left(vec(\mathbf{X}-\tilde{\mathbf{X}})\right)\left(vec(\mathbf{X}-\tilde{\mathbf{X}})\right)^{\mathrm{T}}\right]\ \in\mathbb{R}^{NP\times NP}, \tag{9}$$

having taken (7) into account, the covariance matrix $\mathbf{C}(\mathbf{Y})$ can be given by

$$\mathbf{C}(\mathbf{Y})\approx\frac{d\mathbf{F}}{d\mathbf{X}}\mathbf{C}(\mathbf{X})\left(\frac{d\mathbf{F}}{d\mathbf{X}}\right)^{\mathrm{T}}, \tag{10}$$

where the derivative's matrix is described by formula (6).

Equation (10) presents the generalized law of uncertainty propagation [4]. For the multidimensional measured variable, the covariance matrix contains variances of the successive variables on its principal diagonal, while the elements lying out of the principal diagonal are the variable's covariances. If the particular variables are independent, the covariance matrix takes the form of a diagonal matrix.

3. UNCERTAINTY PROPAGATION IN THE MULTI-CHANNEL SAMPLING TRANSDUCER

In a multi-channel sampling transducer three basic stages of signal processing can be identified [3]:

- Pre-processing and conditioning of analog input signals.
- Sampling and quantization operation.
- Determining of output quantity values using the algorithm for processing of digitised data records.

All the mentioned stages of signal processing are the source of the combined uncertainty of the final measurement result that can be described by Eq. (10). The influence of signal pre-processing has been discussed in [3]. In presented paper one has focused on the analysis of uncertainty propagation related to sampling and quantization of time variable signals by the measurement algorithm.

For the measurement of time variable quantities, the processing result is the function of input quantities and time, because both input quantities and quantization process are time-dependent. In a one-channel transducer, the time variability of the measured value and the influence of the uncertainty connected with the timing jitter between adjacent samples can be presented as a vector function

$$\mathbf{y} = \mathbf{f}(\mathbf{x}, \mathbf{t}), \tag{11}$$

where $\mathbf{x} = \mathbf{g}(\mathbf{t})$ is a vector containing values of time dependent input quantities and $\mathbf{t}^{\mathbf{T}} = [t_1, t_2, \ldots, t_n, \ldots, t_N]$ is a vector containing values of the successive sampling instants.

The time value is determined with the indirect method by summing up the successive intervals between sampling instants. For any sampling instant t_n it can be described as

$$t_n = \tau_1 + \tau_2 + \ldots + \tau_n = \mathrm{f}(\boldsymbol{\tau}), \tag{12}$$

where $\boldsymbol{\tau}^{\mathbf{T}} = [\tau_1, \tau_2, \ldots, \tau_n, \ldots, \tau_N]$.

For uniform sampling, values of the successive time intervals τ_n are the same nominal values corresponding to the sampling period $\tilde{\tau}$. Due to the properties of the real sampling and hold circuit and the uncertainty of the sampling clock generator, the sampling period values are also time dependent, i.e. they depend on the successive sampling instants. Hence

$$\mathbf{t} = \boldsymbol{\Xi}\boldsymbol{\tau} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix} \boldsymbol{\tau}. \tag{13}$$

For a multi-channel transducer, instability of phase relations between particular transducer channels should be additionally taken into account. Then, the output matrix can be written as a function of matrix variable

$$\mathbf{Y} = \mathbf{F}(\mathbf{X}, \mathbf{T}), \tag{14}$$

where $\mathbf{X} = \mathbf{G}(\mathbf{T})$ is a $N \times P$ - dimensional matrix containing in its columns values of time dependent input quantities (1), and $\mathbf{T} = [\mathbf{t}_1 \;\; \mathbf{t}_2 \;\; \cdots \;\; \mathbf{t}_i \;\; \cdots \;\; \mathbf{t}_P] \in \mathbb{R}^{N \times P}$ is a matrix containing in its columns values of the successive sampling instants of a P - channel sampling transducer.

In the real measurement circuit the input signals of the sampling transducer are not available for a direct measurement. The values of these signals can be evaluated on the basis of equations (11) or (14) describing the operations realised by the processing algorithm as well as equations showing the functioning of the signal pre-processing circuit [3]. The input signal uncertainty of the transducer can be determined on the grounds of the covariance matrix of the digitised output signals of the analog-to-digital converters. The values of matrix elements can be estimated during the calibration procedure or they can be based on the known parameters of processing channels elements of the sampling transducer.

In a one-channel transducer the processing algorithm is described by the vector Eq. (11) for which the uncertainty propagation law is as follows

$$\mathbf{C}(\mathbf{y}) \approx \begin{bmatrix} \frac{\partial \mathbf{f}}{\partial \mathbf{x}} & \frac{d\mathbf{f}}{d\mathbf{t}} \end{bmatrix} \begin{bmatrix} \mathbf{C}(\mathbf{x}) & \mathbf{C}(\mathbf{x},\mathbf{t}) \\ \mathbf{C}(\mathbf{t},\mathbf{x}) & \mathbf{C}(\mathbf{t}) \end{bmatrix} \begin{bmatrix} \left(\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right)^{\mathrm{T}} \\ \left(\frac{d\mathbf{f}}{d\mathbf{t}}\right)^{\mathrm{T}} \end{bmatrix} \in \mathbb{R}^{R\times R}. \tag{15}$$

Having taken function (11) into consideration, the derivative occurring in Eq. (15) with respect to the time vector **t** can be modelled by

$$\frac{d\mathbf{f}}{d\mathbf{t}} = \frac{\partial \mathbf{f}}{\partial \mathbf{g}} \frac{d\mathbf{g}}{d\mathbf{t}} + \frac{\partial \mathbf{f}}{\partial \mathbf{t}}. \tag{16}$$

As the measured quantity is not time but the intervals between the successive sampling instants determined with values of the vector $\boldsymbol{\tau}$, having considered that for uniform sampling on the basis of (13)

$$\frac{d\mathbf{t}}{d\boldsymbol{\tau}} = \boldsymbol{\Xi}, \tag{17}$$

$$\mathbf{C}(\mathbf{t}) = \frac{d\mathbf{t}}{d\boldsymbol{\tau}} \mathbf{C}(\boldsymbol{\tau}) \left(\frac{d\mathbf{t}}{d\boldsymbol{\tau}}\right)^{\mathrm{T}} \tag{18}$$

and the Eq. (15) can be converted into

$$\mathbf{C}(\mathbf{y}) \approx \begin{bmatrix} \frac{\partial \mathbf{f}}{\partial \mathbf{x}} & \frac{d\mathbf{f}}{d\boldsymbol{\tau}} \end{bmatrix} \begin{bmatrix} \mathbf{C}(\mathbf{x}) & \mathbf{C}(\mathbf{x},\boldsymbol{\tau}) \\ \mathbf{C}(\boldsymbol{\tau},\mathbf{x}) & \mathbf{C}(\boldsymbol{\tau}) \end{bmatrix} \begin{bmatrix} \left(\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right)^{\mathrm{T}} \\ \left(\frac{d\mathbf{f}}{d\boldsymbol{\tau}}\right)^{\mathrm{T}} \end{bmatrix} \in \mathbb{R}^{R\times R}. \tag{19}$$

Assuming that errors related to the successive sampling intervals are the realizations of the zero-mean independent random variables with variance σ_τ^2, Eq. (18) can be given by

$$\mathbf{C}(\mathbf{t}) = \boldsymbol{\Xi} \sigma_\tau^2 \mathbf{I}_N \boldsymbol{\Xi}^{\mathrm{T}} = \sigma_\tau^2 \boldsymbol{\Xi}\boldsymbol{\Xi}^{\mathrm{T}} = \sigma_\tau^2 \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 2 & \cdots & 2 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 2 & \cdots & N \end{bmatrix}. \tag{20}$$

Respectively, for the matrix function (14) describing the processing algorithm in the multi-channel transducer, the uncertainty propagation law is written as follows

$$\mathbf{C}(\mathbf{Y}) \approx \begin{bmatrix} \frac{\partial \mathbf{F}}{\partial \mathbf{X}} & \frac{d\mathbf{F}}{d\mathbf{T}} \end{bmatrix} \begin{bmatrix} \mathbf{C}(\mathbf{X}) & \mathbf{C}(\mathbf{X},\mathbf{T}) \\ \mathbf{C}(\mathbf{T},\mathbf{X}) & \mathbf{C}(\mathbf{T}) \end{bmatrix} \begin{bmatrix} \left(\frac{\partial \mathbf{F}}{\partial \mathbf{X}}\right)^{\mathbf{T}} \\ \left(\frac{d\mathbf{F}}{d\mathbf{T}}\right)^{\mathbf{T}} \end{bmatrix} \in \mathbb{R}^{RP \times RP}, \tag{21}$$

where elements of submatrices $\frac{\partial \mathbf{F}}{\partial \mathbf{X}}, \frac{d\mathbf{F}}{d\mathbf{T}}$ are the sensitivity coefficients of the output quantities with respect to the successive samples of input quantities and the successive sampling instants, and the partitioned covariance matrix of the independent variable of Eq. (14) that occurs in this equation

$$\mathbf{C}([\mathbf{X} \ \ \mathbf{T}]) = \begin{bmatrix} \mathbf{C}(\mathbf{X}) & \mathbf{C}(\mathbf{X},\mathbf{T}) \\ \mathbf{C}(\mathbf{T},\mathbf{X}) & \mathbf{C}(\mathbf{T}) \end{bmatrix} \in \mathbb{R}^{2NP \times 2NP}, \tag{22}$$

consists of four blocks corresponding to the variance and covariance for the scalar function. Submatrices lying on the principal diagonal of matrix (22) – **C**(**X**) and **C**(**T**) - correspond to variances of matrix variables **X** and **T**, respectively. Whereas, submatrices lying out of the principal diagonal – **C**(**X**, **T**) and **C**(**T**, **X**) - correspond to covariances between these variables. Like for the classical covariance matrix, matrix (22) is the block-symmetric matrix , for which

$$\mathbf{C}(\mathbf{X},\mathbf{T}) = \mathbf{C}(\mathbf{T},\mathbf{X})^{\mathbf{T}} \in \mathbb{R}^{NP \times NP}. \tag{23}$$

Having considered (13), the form of matrix **T** and the timing jitter in P channels of the sampling transducer, matrix $\boldsymbol{\Psi} = [\boldsymbol{\tau}_1, \boldsymbol{\tau}_2, \ldots, \boldsymbol{\tau}_i, \ldots, \boldsymbol{\tau}_p]$ containing values of all $N \times P$ intervals can be defined. Then

$$\frac{d\mathbf{T}}{d\boldsymbol{\Psi}} = \mathbf{I}_P \otimes \boldsymbol{\Xi} \ \in \mathbb{R}^{NP \times NP}, \tag{24}$$

$$\mathbf{C}(\mathbf{T}) = \frac{d\mathbf{T}}{d\boldsymbol{\Psi}} \mathbf{C}(\boldsymbol{\Psi}) \left(\frac{d\mathbf{T}}{d\boldsymbol{\Psi}}\right)^{\mathbf{T}}, \tag{25}$$

where $\otimes$ denotes the Kronecker product [4-5].

If additionally the assumption is satisfied that the random variables in the successive measurements and in addition between the sampling transducer's channels are independent,

$$\mathbf{C}(\boldsymbol{\Psi}) = \mathrm{diag}(\sigma_{\tau 1}^2 \mathbf{I}_N, \sigma_{\tau 2}^2 \mathbf{I}_N, \ldots, \sigma_{\tau i}^2 \mathbf{I}_N, \ldots, \sigma_{\tau P}^2 \mathbf{I}_N), \tag{26}$$

and having taken (24) into consideration

$$\mathbf{C}(\mathbf{T}) = \text{diag}(\sigma_{\tau 1}^2, \sigma_{\tau 2}^2, \ldots, \sigma_{\tau i}^2, \ldots, \sigma_{\tau P}^2) \otimes \boldsymbol{\Xi}\boldsymbol{\Xi}^{\mathbf{T}} . \tag{27}$$

The two-channel transducer realising an algorithm of impedance components measurement will be presented as an example of the calculation method of the sensitivity coefficients occurring in Eq. (21).

4. APPLICATION EXAMPLE

A two-channel sampling transducer with proper input signal pre-processing circuits together with a sinusoidal excitation generator can be used to measure impedance components [3, 6-10].

One of the frequently used algorithms for the determination of impedance components is the indirect method, in which on the grounds of the estimates of orthogonal components of voltage and current amplitudes, the values of impedance components in the Cartesian co-ordinate system can be calculated

$$R = \frac{U_c I_c + U_s I_s}{I_c^2 + I_s^2}, \qquad X = \frac{U_s I_c - U_c I_s}{I_c^2 + I_s^2}, \tag{28}$$

and to determine the components values of voltage and current amplitudes the discrete Fourier transform is used. It is given by [8]

$$\mathbf{Y} = \frac{2}{N} \mathbf{A}^{\mathbf{T}} \mathbf{X}, \tag{29}$$

where: $\mathbf{Y} = [\mathbf{y}_i \ \ \mathbf{y}_u] = \begin{bmatrix} I_c & U_c \\ I_s & U_s \end{bmatrix}$, $\mathbf{X} = [\mathbf{i} \ \ \mathbf{u}] \in \mathbb{R}^{N \times 2}$,

$\mathbf{A}^{\mathbf{T}} = [\mathbf{a}^{\mathbf{T}}(1) \ \ \mathbf{a}^{\mathbf{T}}(2) \ \cdots \ \mathbf{a}^{\mathbf{T}}(n) \ \cdots \ \mathbf{a}^{\mathbf{T}}(N)] \in \mathbb{R}^{2 \times N}$,

$\mathbf{a}(n) = [\sin n\omega_g \tau, \cos n\omega_g \tau]$.

The value of scaling factor 2/N in Eq. (29) results from the definition of coefficients of discrete Fourier series.

To simplify the consideration it has been assumed that the sampling process is synchronous with the angular frequency ω_g of the forcing generator, and the uncertainties related to the synchronisation process can be ignored.

The processing Eq. (29) is a function of the input variable **X** and the matrix of $N \times 2$ sampling instants **T**, for which according to the formula (21), the covariance matrix of voltage and current components **C**(**Y**) can be calculated. The derivatives found in Eq. (21) are given by formulae

$$\frac{d\mathbf{Y}}{d\mathbf{X}} = \frac{2}{N} \left(\mathbf{I}_2 \otimes \mathbf{A}^{\mathbf{T}} \right) \in \mathbb{R}^{4 \times 2N}, \tag{30}$$

$$\frac{d\mathbf{Y}}{d\mathbf{T}} = \frac{\partial \mathbf{Y}}{\partial \mathbf{A}^{\mathbf{T}}} \frac{\partial \mathbf{A}^{\mathbf{T}}}{\partial \mathbf{A}} \frac{\partial \mathbf{A}}{\partial \mathbf{T}} + \frac{\partial \mathbf{Y}}{\partial \mathbf{X}} \frac{\partial \mathbf{X}}{\partial \mathbf{T}} \in \mathbb{R}^{4 \times 2N}, \tag{31}$$

$$\frac{d\mathbf{Y}}{d\mathbf{T}} = \frac{2}{N}\mathbf{K}_{22}(\mathbf{I}_2 \otimes \mathbf{X}^{\mathbf{T}})\frac{\partial \mathbf{A}}{\partial \mathbf{T}} + \frac{2}{N}\left(\mathbf{I}_2 \otimes \mathbf{A}^{\mathbf{T}}\right)\frac{\partial \mathbf{X}}{\partial \mathbf{T}}, \tag{32}$$

where $\mathbf{K}_{22}$ is 4×4 - dimensional commutation matrix [4-5], and $2N\times 2N$ - dimensional matrices of partial derivatives

$$\frac{\partial \mathbf{A}}{\partial \mathbf{T}} = \omega_g \begin{bmatrix} 1 \\ 1 \end{bmatrix} \otimes \operatorname{diag}(\mathbf{a}(1), \mathbf{a}(2), \ldots, \mathbf{a}(n), \ldots \mathbf{a}(N))(\mathbf{I}_N \otimes \mathbf{R}_{\pi/2})\mathbf{K}_{N2}, \tag{33}$$

$$\frac{\partial \mathbf{X}}{\partial \mathbf{T}} = \operatorname{diag}\left(\frac{\partial \mathbf{x}_1}{\partial \mathbf{t}_1}, \frac{\partial \mathbf{x}_2}{\partial \mathbf{t}_2}\right). \tag{34}$$

containing the sensitivity coefficients of factors of function (29) with respect to the successive sampling instants. Matrix $\mathbf{K}_{N2}$ occurring in formula (33) is also a commutation matrix, whereas $\mathbf{R}_{\pi/2}$ is a $\pi/2$ angle rotation matrix [8].

The value of matrix elements (33) can be obtained before the measurement starts, after the generator's angular frequency ω_g and the sampling period τ are defined. Values of derivatives occurring in matrix (34) can be calculated experimentally during the execution of the measurement by determining the values of increments of the input quantity between successive sampling instants. As the signals measured in the analysed example are sinusoidal, and the processing results are the coefficients values of a model of these signals given by

$$\mathbf{X} = \mathbf{A}\mathbf{Y}, \tag{35}$$

the values of the sensitivity coefficients (34) can be obtained more precisely as

$$\frac{\partial \mathbf{X}}{\partial \mathbf{T}} = \frac{\partial \mathbf{X}}{\partial \mathbf{A}}\frac{\partial \mathbf{A}}{\partial \mathbf{T}}, \tag{36}$$

where $\frac{\partial \mathbf{X}}{\partial \mathbf{A}} = \mathbf{Y}^{\mathbf{T}} \otimes \mathbf{I}_N$.

Hence finally

$$\frac{d\mathbf{Y}}{d\mathbf{T}} = \frac{2}{N}\left(\mathbf{K}_{22}(\mathbf{I}_2 \otimes \mathbf{X}^{\mathbf{T}}) + (\mathbf{Y}^{\mathbf{T}} \otimes \mathbf{A}^{\mathbf{T}})\right)\frac{\partial \mathbf{A}}{\partial \mathbf{T}}. \tag{37}$$

In many cases it can be assumed that the covariance matrix (22) is simplified to its block-diagonal form, if matrices (23) are null submatrices. Such a situation happens if the measured quantity $\mathbf{X}$ does not influence the values of time intervals. Then equation (22) can be written in a simplified form

$$\mathbf{C}(\mathbf{Y}) \approx \frac{\partial \mathbf{F}}{\partial \mathbf{X}}\mathbf{C}(\mathbf{X})\left(\frac{\partial \mathbf{F}}{\partial \mathbf{X}}\right)^{\mathbf{T}} + \frac{\partial \mathbf{F}}{\partial \mathbf{T}}\mathbf{C}(\mathbf{T})\left(\frac{\partial \mathbf{F}}{\partial \mathbf{X}}\right)^{\mathbf{T}} = \mathbf{C}_{\mathbf{X}}(\mathbf{Y}) + \mathbf{C}_{\mathbf{T}}(\mathbf{Y}), \tag{38}$$

containing two components: time independent $\mathbf{C_X(Y)}$ - related to measurement uncertainty of input quantities **X** and time dependent $\mathbf{C_T(Y)}$ - resulting from uncertainty of determining the sampling instants.

If additionally it can be assumed that errors connected with measurements of successive values of measured quantities are the realizations of the zero-mean independent random variables with identical variances σ_x^2 in both transducer channels, and $\sigma_{\tau 1}^2 = \sigma_{\tau 2}^2 = \sigma_\tau^2$, then

$$\mathbf{C_X}(\mathbf{Y}) = \frac{2}{N}\sigma_x^2 \mathbf{I}_2, \tag{39}$$

$$\mathbf{C_T}(\mathbf{Y}) = \frac{8\sigma_\tau^2}{N^2}\begin{pmatrix} \mathbf{K}_{22}(\mathbf{I}_2 \otimes \mathbf{X}^\mathbf{T})\mathbf{P}(\mathbf{I}_2 \otimes \mathbf{X})\mathbf{K}_{22} + \\ +(\mathbf{Y}^\mathbf{T} \otimes \mathbf{A}^\mathbf{T})\mathbf{P}(\mathbf{Y} \otimes \mathbf{A}) + 2(\mathbf{Y}^\mathbf{T} \otimes \mathbf{A}^\mathbf{T})\mathbf{P}(\mathbf{I}_2 \otimes \mathbf{X})\mathbf{K}_{22} \end{pmatrix}, \tag{40}$$

where $\mathbf{P} = \frac{\partial \mathbf{A}}{\partial \mathbf{T}} \mathbf{\Xi}\mathbf{\Xi}^\mathbf{T} \left(\frac{\partial \mathbf{A}}{\partial \mathbf{T}}\right)^\mathbf{T}$.

The covariance matrix (40) contains three components weighted by matrix **P**, which depends on the form of the processing algorithm . The first component considers the influence of the measured quantity **X** on the uncertainty of the processing result , the second one results from the calculation method of the matrix of sensitivity coefficients (36). The third component with a double value contains components of the covariance matrix that arise from the simultaneous influence of both these sources of uncertainty.

To determine the uncertainty of impedance components, the obtained covariance matrix **C**(**Y**) needs to be used, applying the uncertainty propagation law for formulae (28) [3].

5. CONCLUSIONS

The method for determining of the processing uncertainty in a multi-channel sampling transducer presented in this paper ensures that evaluations of combined uncertainty of measurement results are obtained for time-variable multidimensional measured quantities. With regard to the matrix notation used in this method, apart from the evaluation of variances for the particular variables, all the covariance evaluations collected in the covariance matrix are also determined. The application of the generalised law of uncertainty propagation makes it possible to consider all the essential sources of uncertainty occurring in the multi-channel sampling transducer, including the influence of timing jitter between the successive samples in the separate processing channel, as well as the cases of interdependence between all the variables occurring in the particular processing channels.

The covariance matrix notation in the form of the expression containing sensitivity coefficients can be used to minimize the measurement uncertainty by identification of the most essential components of the combined processing uncertainty.

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OCENA NIEPEWNOŚCI ALGORYTMU PRZETWARZANIA WIELKOSCI ZMIENNYCH W CZASIE W WIELOKANAŁOWYCH SYSTEMACH POMIAROWYCH

Streszczenie

W pracy przedstawiono możliwość wykorzystania uogólnionego prawa propagacji niepewności (10) do wyznaczenia złożonej niepewności standardowej *P*-kanałowego przetwornika próbkującego. Algorytm przetwarzania dla takiego przetwornika można w ogólnym przypadku opisać funkcją macierzową zmiennej macierzowej, gdyż wynikiem przetwarzania jest wektor lub macierz wielkości wyjściowych. Jeżeli wielkości mierzone są zmienne w czasie, wynik przetwarzania jest również zależny od czasu, co można opisać funkcją (14). Wartość czasu jest określana metodą pośrednią poprzez sumowanie kolejnych odcinków czasu pomiędzy chwilami próbkowania (12). W przypadku próbkowania równomiernego wartości elementów wektora czasu, są określane metodą zliczania kolejnych wartości okresu próbkowania o nominalnie jednakowej wartości $\tilde{\tau}$. Dla przetwornika jednokanałowego można to przedstawić w postaci funkcji (13). Niepewność związaną z niestałością związków fazowych pomiędzy kolejnymi chwilami próbkowania dla przetwornika wielokanałowego zebranymi w macierzy **T**, można przedstawić w postaci macierzy kowariancji (25). Złożona niepewność standardowa przetwornika próbkującego wynikająca z propagacji niepewności związanych z wielkością mierzoną **X** oraz niestałością związków fazowych pomiędzy próbkami i kanałami przetwornika **T** opisuje wtedy wzór (21).

Wykorzystanie tej metody do oceny niepewności przetwarzania pokazano na przykładzie układu do pomiaru składowych impedancji, w którym jest realizowany algorytm opisany równaniem (29). Uwzględniając postać sygnału wejściowego, współczynniki wrażliwości wielkości wyjściowej algorytmu **Y** można wyznaczyć ze wzorów (30)-(34). Po przyjęciu pewnych założeń upraszczających, dotyczących właściwości zmiennych losowych związanych z błędami procesu próbkowania i kwantowania sygnałów, złożoną niepewność standardową algorytmu (29) opisują równania (38)-(40).